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(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIRST SEMESTER EXAMINATION, DECEMBER 2015

FIRST YEAR [BATCH 2015-18]

MATHEMATICS [Gen]

Paper : I

Date : 22/12/2015

Time : 11 am – 2 pm

Full Marks : 75

[Use a separate Answer Book for each Group]

Group - A

(*Answer* ***any five*** *questions*)

1. Show that the equation $\tan\left(i\log\frac{x-iy}{x+iy}\right)=2$ represents the hyperbola $x^2-y^2=xy$. [5]

2. a) If α,β,γ be the roots of the equation $2x^3+x^2+x+1=0$, find the equation whose roots are $\frac{1+\alpha}{1-\alpha},\frac{1+\beta}{1-\beta},\frac{1+\gamma}{1-\gamma}$. [3]

   b) Remove the second term of the equation $x^3+6x^2+9x+4=0$. [2]

3. Solve: $x^3+12x-12=0$. [5]

4. a) For what value of λ, the matrix $\begin{pmatrix}2&3&1\\3&\lambda&0\\2&5&2\end{pmatrix}$ is singular. [2]

   b) Prove that $\begin{vmatrix}a+b+c&-c&-b\\-c&a+b+c&-a\\-b&-a&a+b+c\end{vmatrix}=2(a+b)(b+c)(c+a)$. [3]

5. Solve by Cramer's rule :

$$\begin{aligned}x+y+z&=2\\x-y+2z&=3\\3x+5y-7z&=-4\end{aligned}.$$ [5]

6. Compute the adjoint and the inverse of $A=\begin{bmatrix}1&2&3\\3&-2&1\\4&2&1\end{bmatrix}$. [5]

7. Show that $A=\frac{1}{3}\begin{bmatrix}-1&2&-2\\-2&1&2\\2&2&1\end{bmatrix}$ is orthogonal. Hence find A^{-1}. [4+1]

8. Solve, if possible, the following system of equations :

$$\begin{aligned}x+2y+3z&=2\\2x+4y+2z&=-1.\\x+2y-2z&=5\end{aligned}$$ [5]

Group - B

(*Answer **any five** questions from **each unit***)

Unit - I

9. a) Given $f:\mathbb{R}\to\mathbb{R}$ is defined by $f(x) = x^2,\ x\in\mathbb{R}$. Test whether f is injective or surjective. [2]

 b) $f:\mathbb{Z}\to\mathbb{Q}$ is defined by $f(x)=\frac{x}{2},\ x\in\mathbb{Z}$ and $g:\mathbb{Q}\to\mathbb{Q}$ is defined by $g(x) = x^2,\ x\in\mathbb{Q}$. Find gof(x) and fog(x), if exist. [3]

10. Prove that if $x^2 = e$ for all elements x of a group G, then G is commutative. (e: identity element of G). [5]

11. Prove that $(2\mathbb{Z},+,\bullet)$ is a commutative ring. Is it a ring with unity? ($\mathbb{Z}$ is the set of all integers). [4+1]

12. Prove that the ring $\{a+b\sqrt{2}:a,b\in\mathbb{Q}\}$ forms a field. [5]

13. State the replacement theorem. Find a basis of the real vector space $\mathbb{R}^3$ containing the vectors (1,2,0), (1,3,1). [1+4]

14. For the real quadratic form $x_1^2+x_2^2-x_3^2+2x_1x_2+2x_1x_3-2x_2x_3$ find the associated matrix and hence find it's nature. [5]

15. Find the eigen values and corresponding eigen vectors of $A=\begin{pmatrix}1 & 3\\ 4 & 5\end{pmatrix}$. [5]

16. State Cayley-Hamilton theorem and use it to compute the inverse of the matrix $\begin{bmatrix}2 & 1\\ 3 & 5\end{bmatrix}$. [2+3]

Unit - II

17. Find the value of β for which the sum of squares of the roots of the equation $x^2+(2-\beta)x+1-\beta=0$ has a minimum value. [5]

18. If $y=\frac{\sin^{-1}x}{\sqrt{1-x^2}},|x|<1$, show that $(1-x^2)y_{n+2}-(2n+3)xy_{n+1}-(n+1)^2y_n=0$. [5]

19. Show that the function $f(x,y)=\sin(x^2+y^2)$ is continuous at any point (a, b) in $\mathbb{R}^2$. [5]

20. If u = f(x,y) and $x=r\cos\theta, y=r\sin\theta$, use the chain rule to prove that $u_x^2+u_y^2=u_r^2+\frac{1}{r^2}u_\theta^2$. [5]

21. State Schwarz's theorem on mixed second order partial derivatives for two variables function. If $u=\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sin 2u$. [2+3]

22. Prove that the curves $\frac{x^2}{a}+\frac{y^2}{b}=1$ and $\frac{x^2}{A}+\frac{y^2}{B}=1$ will cut orthogonally if $a-b=A-B$. [5]

23. Show that the Pedal equation of the astroid $x^{2/3}+y^{2/3}=a^{2/3}$ is $r^2+3p^2=a^2$. [5]

24. Find the radius of curvature at (r,θ) on the cardioide $r=a(1-\cos\theta)$, and show that it varies as $\sqrt{r}$. [4+1]

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